

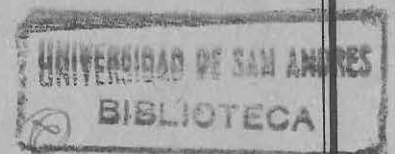
CICLO DE SEMINARIOS 1996  
DEPARTAMENTO DE ECONOMIA

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**Intergovernmental  
Transfers, Wages and  
Employment in the Private  
and Public Sectors.**

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**Oswaldo H. Schenone**



**Universidad de  
San Andrés**

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Public Sectors.**

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Intergovernmental Transfers, Wages and  
Employment in the Private and Public Sectors<sup>1</sup>

Osvaldo H. Schenone

Universidad de San Andrés

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I. Introduction

This paper is concerned with the effects of fiscal transfers between jurisdictions upon wages and the allocation of labor in the private and public sectors in each jurisdiction. Such transfers are assumed to finance public employment in the receiving jurisdiction.

To highlight the effects on the allocation of employment, it will be assumed that public expenditure in each jurisdiction consists only of employment to produce a public good, available to the private sector at zero marginal cost, which in turn increases the productivity of labor in the private sector within the respective jurisdiction<sup>2</sup>. This productive role of governments follows R. Findlay's hypothesis "...that public expenditure on public goods --administration, law and order, roads, justice, and so on-- acts as an externality to private economic activities,

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<sup>1</sup> Comments by P. Beker are gratefully acknowledged.

<sup>2</sup> The public sector is viewed as an entity which bids away labor from the private sector and, in return, gives back a public good which increases the productivity of the workers left in the private sector. This view may be consistent with the "safety net" approach to public employment, if one is prepared to argue that the prevention of social disruption increases the productivity of the labor left to work in the private sector.



enhancing the private outputs from private inputs." (Findlay, 1991).

One jurisdiction is assumed to receive from the other a fixed amount to be spent on public employment without imposing any tax on its own residents. For convenience, this jurisdiction might be called "poor".

The other jurisdiction is assumed to collect enough tax revenues to perform the transfer and to hire workers to produce its own public good. Such revenue is collected by taxing employment in the private sector. Accordingly, this jurisdiction might be called "rich".

It will also be assumed that the wage (both public and private) in the rich jurisdiction affects positively the productivity of private employment in the poor jurisdiction. This assumption creates a trade off for the poor jurisdiction<sup>3</sup>: The cost of getting higher transfers is, *ceteris paribus*, the reduction in the wage in the rich jurisdiction, hence a reduction in its own private labor's productivity.

Labor is not mobile across jurisdictions but it is mobile

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<sup>3</sup> This assumption also conveys the idea that the prosperity of the rich jurisdiction spills over to the private sector in the poor one. One can think of this as a consequence of the residents in the rich jurisdiction being the bulk of the customers of the private sector in the poor jurisdiction.

between the private and public sectors in each jurisdiction. Labor supply in each one is given independently of the wage rate in the respective jurisdiction.

This setup resembles the situation of several provincial governments in Argentina whose revenues are essentially transfers from other provinces and the expenses consists mainly of wages. At the same time the fortunes of the private sector in such provinces depend ultimately on the well being of the transfer-paying jurisdictions.

The questions to be answered are: What are the effects on wages, the quantity of the public good, and employment in each sector in each jurisdiction of increasing the taxes raised in the rich jurisdiction, holding the transfers constant? What are the effects on the same variables of increasing the transfers to the poor jurisdiction holding the taxes imposed by the rich jurisdiction constant?

To ask the questions in a more policy oriented fashion, does the creation of public jobs in one jurisdiction promote more or less public employment in the other? What about private employment in each jurisdiction? Do additional transfers to the poor jurisdiction increase the wage rate in such jurisdiction? Does a policy which increases wages in one jurisdiction also increases (or decreases) wages in the other jurisdiction?

The answers to these questions will, obviously, depend on the conditions under which these changes take place, and the paper explores the results corresponding to several alternatives. A basic model is presented in the next section. The model is expanded in section III by explicitly assuming a policy maker's objective function that depends positively on public employment in each jurisdiction.

## II. A simple model

The labor market equilibrium in the rich jurisdiction requires the marginal productivity of labor in the private sector,  $F^L$ , to be equal to the take home wage,  $w$ , plus the tax on private employment,  $T$ .

$$(1) \quad F^L(L^P, X) - T - w = 0,$$

where  $L^P$  represents the number of workers employed in the private sector and  $X$  stands for the quantity of the public good produced in the rich jurisdiction. It will be assumed that the signs of the derivatives,  $F^{LL}$  and  $F^{LX}$ , are negative and positive respectively.

The total labor available in the rich jurisdiction,  $L$ , is allocated between private and public employment,  $L^G$ . Accordingly,

$$(2) \quad L^P + L^G - L = 0.$$

Tax revenue,  $T L^P$ , is spent on (a) hiring labor to produce  $X$  or (b) making transfers to the poor jurisdiction, where the



resources are spent on hiring labor in the quantity  $N^G$  at the wage rate  $w^*$ . Thus,

$$(3) \quad L^G w + N^G w^* - T L^P = 0.$$

The public good  $X$  is produced according to the production function

$$(4) \quad G(L^G) = X,$$

with  $G'$ , the first derivative of the function  $G$ , positive.

In the poor jurisdiction the marginal productivity of labor in the private sector,  $R^N$ , equals the take home wage rate as no taxes are imposed. It is further assumed that  $R^N$  depends positively on  $w$ , a proxy for prosperity in the rich jurisdiction. Accordingly,

$$(5) \quad R^N(N^P, Y, w) - w^* = 0,$$

where  $N^P$  stands for the quantity of labor employed in the private sector in the poor jurisdiction and  $Y$  represents the quantity of the public good produced in the poor jurisdiction. The signs of the derivatives of the function  $R^N$  are assumed to be

$$R^{NN} < 0; R^{NY} > 0; \text{ and } R^{Nw} > 0.$$

The total supply of labor is  $N$ , so that

$$(6) \quad N^P + N^G - N = 0.$$

The total resources available to spent on public employment,  $K$ , is given. Hence,

$$(7) \quad w^* N^G - K = 0.$$

The production of the public good takes place according to the production function

$$(8) \quad S(N^G) = Y,$$

with  $S'$ , the first derivative of the function  $S$ , positive.

Replacing equations (2) and (4) into (1), and equations (6) and (8) into (5) gives a system of four equations in four unknowns,  $L^G$ ,  $w$ ,  $N^G$ , and  $w^*$ :

$$(9) \quad F^L(L-L^G, G(L^G)) - T - w = 0,$$

$$(10) \quad L^G w + N^G w^* - T(L-L^G) = 0,$$

$$(11) \quad R^N(N-N^G, S(N^G), w) - w^* = 0,$$

$$(12) \quad w^* N^G - K = 0.$$

The determinant of first derivatives of the system above,  $\Delta$ , is positive:

$$\Delta = (-F^{LL} + F^{LX} G') \{N^G(-R^{NN} + R^{NY} S') + w^*\} L^G + (w+T) \{N^G(-R^{NN} + R^{NY} S') + w^*\} > 0.$$

Hence, by virtue of the implicit function theorem, the values of the variables that satisfy equations (9) to (12) can be written as follows

$$(9') \quad L^G = L^G(T, K)$$

$$(10') \quad w = w(T, K)$$

$$(11') \quad N^G = N^G(T, K)$$

$$(12') \quad w^* = w^*(T, K).$$

Having set the scenario, it is now possible to look for the answers to the questions set out at the beginning of the paper.



The effects of increasing taxes

holding transfers constant

If the rich jurisdiction increases its taxes without changing the transfers to the poor jurisdiction, public employment and the quantity produced of the public good are bound to raise in the rich jurisdiction. Thus private employment has to yield, which it does under the influence of higher taxation.

$$\frac{\partial L^G}{\partial T} = \frac{L}{\Delta} [(-R^{NN} + R^{NY}S')N^{G+W*}] > 0 \quad (13)$$

$$\frac{\partial X}{\partial T} = G' \frac{dL^G}{dT} > 0$$

$$\frac{\partial L^P}{\partial T} - \frac{\partial L^G}{\partial T} < 0$$

The take home wage rate, however, may go up or down, and the reason is that several forces are operating in possibly opposite directions: While the increased quantity of the public good (made available by the correspondingly higher public employment) increases the private demand for labor, the increased tax rate reduces the quantity privately demanded of labor. The net result may be either a positive or a negative excess demand for labor at the previously prevailing  $w$ ; the overall effect on the take home wage is ambiguous.

$$\frac{\partial w}{\partial T} = \frac{1}{\Delta} [(-R^{NN} + R^{NY}S')N^{G+W*}] [L^P(-F^L + F^{LX}G') - F^L] \quad (14)$$



Since the first bracket in equation (14) is unambiguously positive, the ambiguity in the sign of such equation stems from the second bracket which can be positive or negative. It will be negative; that is, the take home wage will fall as a consequence of the higher taxation, if and only if

$$0 < -E_{LL} + E_{LX} (L^P G'/G) = \beta < 1,$$

where  $E_{LL}$  and  $E_{LX}$  stand for the elasticities of the marginal productivity of labor with respect to the quantity of labor and the quantity of the public good, respectively.

The ambiguity is not only due to the effect of the public good upon the demand for labor: Even if  $F^{LX}$  was zero (hence  $E_{LX} = 0$ ),  $\beta$  can still be greater or smaller than one depending on the wage elasticity of the demand for labor. An elasticity lower than one will entail  $\beta > 1$  so that  $w$  can rise in response to higher taxation even if there was no effect of the public good upon the demand for labor.<sup>4</sup>

Accordingly, the prosperity spillover effect on the productivity of private labor in the poor jurisdiction is also ambiguous, so that private and public employment there can remain constant or move in either direction, and so can the wage rate,

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<sup>4</sup> The reason for this result is that a sufficiently inelastic private demand for labor may allow the government to collect enough revenue to increase its labor force by more than the corresponding reduction in private employment. Thus an excess demand for labor at the originally prevailing take home wage will arise and  $w$  will quite naturally go up.

despite the fact that the transfers remain unchanged. Of course, the changes in the wage rate are inversely proportional to the changes in public employment, so that the government budget constraint is satisfied<sup>5</sup>.

$$\frac{\partial N^G}{\partial T} = \frac{N^G}{\Delta} R^{NW} [F^L - L^P (-F^{\kappa} + F^{LX} G')] \quad (15)$$

The expression above is positive if  $\beta < 1$ ; that is, public employment in the poor jurisdiction will automatically raise (and private employment will fall accordingly) if the take home wage in the rich jurisdiction falls as a consequence of higher taxation and *vice versa*. Under these circumstances the wage rate in the poor jurisdiction goes down too:

$$\frac{\partial w^*}{\partial T} = -\frac{w^*}{\Delta} R^{NW} [F^L - L^P (-F^{\kappa} + F^{LX} G')] \quad (16)$$

This is clearly negative if  $\beta < 1$  and *vice versa*; that is, the wage rate in the poor jurisdiction and the take home wage rate in the rich jurisdiction move together when taxation changes, *ceteris paribus*.

To summarize: The effects of this policy are to raise public

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<sup>5</sup> This can be verified by noting from equations (15) and (16) that

$$\frac{1}{N^G} \frac{\partial N^G}{\partial T} = -\frac{1}{w^*} \frac{\partial w^*}{\partial T}$$



employment in both jurisdictions and reduce wages everywhere (if  $\beta < 1$ ) or, alternatively, to raise wages everywhere and reduce public employment in the poor jurisdiction (if  $\beta > 1$ ).

The effects of increasing transfers

holding taxes constant

If the transfers to the poor jurisdiction are increased without increasing the tax rate, less resources will be left to the government of the rich jurisdiction. Consequently, public employment and the quantity produced of the public good are bound to fall in the rich jurisdiction, as indicated by equation (17). Both of these results will, in turn, yield an excess supply of labor at the previously prevailing wage rate. Hence the take home wage will fall as indicated by equation (18) and private employment will raise in the rich jurisdiction.

$$\frac{\partial L^G}{\partial K} - \frac{1}{\Delta} [w^* + N^G (-R^{NN} + R^{NY} S')] < 0 \quad (17)$$

$$\frac{\partial X}{\partial K} - G' \frac{\partial L^G}{\partial K} < 0$$

$$\frac{\partial w}{\partial K} - \frac{1}{\Delta} (-F^L + F^{LX} G') [(-R^{NN} + R^{NY} S') N^G + w^*] < 0 \quad (18)$$

$$\frac{\partial L^P}{\partial K} - \frac{\partial L^G}{\partial K} > 0$$

The arrival of additional resources to the poor jurisdiction will get immediately translated into higher public employment as indicated by equation (19), hence higher production of the public good and less private employment.

$$\frac{\partial N^G}{\partial K} = \frac{1}{\Delta} [(-F^{\kappa} + F^{LX}G') (L^G + R^{NN}N^G) + W + T] > 0 \quad (19)$$

$$\frac{\partial Y}{\partial K} - S' \frac{\partial N^G}{\partial K} > 0$$

$$\frac{\partial N^P}{\partial K} - \frac{\partial N^G}{\partial K} < 0$$

A question, however, arises about the wage rate. An excess demand for labor may not develop despite the increase in public employment, as private labor productivity has been enhanced by the greater availability of the public good and, at the same time, has been discouraged by the reduction in  $w$ ; that is, by the prosperity spillover effect in reverse. Thus the wage rate in the poor jurisdiction may go up or down as a result of receiving additional transfers, as indicated by equation (20).

$$\frac{\partial w^*}{\partial K} = \frac{1}{\Delta} [(-R^{NN} + R^{NY}S') L^G - R^{NW}w^*] (-F^{\kappa} + F^{LX}G') + F^L (-R^{NN} + R^{NY}S') \quad (20)$$

Unlike the previous case, public employment in both jurisdictions can never change in the same direction and wages in both jurisdictions may change in opposite directions, as a consequence of changing the amount of transfers *ceteris paribus*.

### III. A policy maker's optimization model

The subject of the preceding section was the response of public employment in each jurisdiction to changes of  $T$  and  $K$  that simply took place, not being a consequence of the maximizing behaviour of a policy maker. This section explores the response of the same variables when they are constrained to maximize the policy maker's objective function in addition to the constraints imposed by equations (9) to (12).

This section will consider a policy maker whose objective function,  $U$ , depends positively on public employment in each jurisdiction or, equivalently, on the production of the public good in each jurisdiction. Hence, the policy maker is assumed to

$$\text{Max } U(L^G, N^G),$$

subject to the restrictions given by equations (9) to (12) which, in turn, can be written as follows: From equations (9), (10) and (12),

$$(21) \quad L^G (F^L - T) = T(L - L^G) - K,$$

and from equations (11) and (12):

$$(22) \quad R^N N^G = K,$$

where, of course, the functions  $F^L$  and  $R^N$  are defined by equations (1) and (5) respectively.

Let  $U_L$  and  $U_N$  be the policy maker's marginal utilities of  $L^G$  and  $N^G$  respectively. The first order conditions for this problem



are given by equations (21) to (24):

$$(23) \quad 0 = U_L - \lambda_1 [F^L + L^G(-F^{LL} + F^{LX} G')] - \lambda_2 R^{HN}(-F^{LL} + F^{LX} G')$$

$$(24) \quad 0 = U_H - \lambda_2 [-R^{HN} + R^{HY} S' + K/(N^G)^2]$$

This is a system of four equations in four variables ( $L^G$ ,  $N^G$ ,  $\lambda_1$  and  $\lambda_2$ ) and two parameters (T and K), where the lambdas are the Lagrange multipliers associated with restrictions (21) and (22).

The determinant of first derivatives of the system above, A, is positive:

$$A = [F^L + L^G(-F^{LL} + F^{LX} G')] [-R^{HN} + R^{HY} S' + K/(N^G)^2]^3 > 0.$$

Hence, equations (21) to (24) yield the optimal values of  $L^G$  and  $N^G$  as functions of T and K,  ${}^0L^G = {}^0L^G(T, K)$  and  ${}^0N^G = {}^0N^G(T, K)$  respectively.

The policy maker's choice of T and K comes from maximizing the indirect utility function

$$U({}^0L^G(T, K), {}^0N^G(T, K)) = V(T, K).$$

The optimal values of T and K are those which satisfy:

$$(\partial {}^0N^G / \partial T) + (\partial {}^0L^G / \partial T) - (\partial {}^0N^G / \partial K) + (\partial {}^0L^G / \partial K) \quad (25)$$

where the partial derivatives must be calculated from equations (21) to (24). These derivatives may not be the same as those calculated from equations (9) to (12) in section II, because an

additional constraint is now being introduced; namely, that the values of  $L^G$  and  $N^G$  are optimally chosen.

The effects of increasing transfers  
holding taxes constant

The signs of the derivatives of the right hand side of equation (25) are now studied.

$$\frac{\partial L^G}{\partial K} = -1 / \left[ -R^{NN} + R^{NY} S' + \frac{K}{N^G} \right] < 0 \quad (26)$$

$$\frac{\partial N^G}{\partial K} = \left[ -R^{NN} + R^{NY} S' + \frac{K}{N^G} \right]^{-2} \left[ (R^{NW} + \frac{L^G}{N^G}) (-F^L + F^{LX} G') + \frac{F^L}{N^G} \right] > 0 \quad (27)$$

As in the preceding section, increasing transfers holding T constant increases public employment in the poor jurisdiction and decreases it in the rich one.

The fall in  $L^G$  implies an increase in  $L^P$  and a fall in  $X = G(L^G)$ . This, in turn, means that there will be an excess supply of labor at the previously prevailing wage rate. Hence the take home wage rate in the rich jurisdiction,  $w$ , will fall if a maximizing policy maker increases  $K$  holding  $T$  constant.

This result will generate a prosperity spillover effect in reverse which reduces the marginal productivity of labor in the poor jurisdiction, contrary to the consequences of both the increase in  $Y = S(N^G)$  and the reduction in  $N^P$ . The net outcome is

ambiguous, and the marginal productivity of labor, hence the wage rate  $w^*$ , may rise or fall in the poor jurisdiction.

Therefore the take home wage rate will unambiguously fall in the rich jurisdiction, and it may even fall everywhere if a maximizing policy maker increases  $K$  holding  $T$  constant.

The effects of increasing taxes  
holding transfers constant

The signs of the derivatives on the left hand side of equation (25) are now studied.

$$\frac{\partial^0 L^G}{\partial T} = \frac{L^G}{N^G} \left[ -R^{NN} + R^{NY} S' + \frac{K}{N^G} \right] > 0 \quad (28)$$

$$\frac{\partial^0 N^G}{\partial T} = \frac{N^G}{R^{NN} + R^{NY} S' + \frac{K}{N^G}} \left[ F^L - L^G \left( -F^{LK} + F^{LX} G' \right) \right] / \left[ -R^{NN} + R^{NY} S' + \frac{K}{N^G} \right]^2 \quad (29)$$

As in the preceding section, increasing taxes holding  $K$  constant unambiguously increases  $L^G$  and has an ambiguous effect on  $N^G$ . The latter is positive if  $\beta < 1$  and *vice versa*.<sup>6</sup> Unlike the previous section, however, this ambiguity can be solved by using equation (25). Since its right hand side is unambiguously negative and  $\partial^0 L^G / \partial T > 0$ , it therefore follows that  $\partial^0 N^G / \partial T < 0$ . A maximizing policy maker could not be in equilibrium if it were still feasible, by changing the parameters at his disposal, to increase both  $L^G$  and

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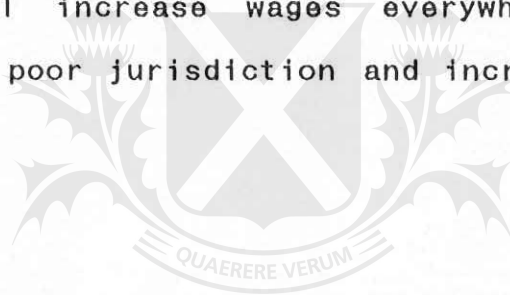
<sup>6</sup> - The absolute values of these effects are, however, not the same as in the preceding section.



$N^G$  hence to further increase  $U$ .<sup>7</sup>

A fall in  $N^G$  must be accompanied by (1) an increase in  $w^*$  (to satisfy the condition  $N^G w^* = K$ , which is held constant); (2) an increase in  $N^P$  for the total labor supply is constant in each jurisdiction; and (3) a fall in  $Y=S(N^G)$ . The only conceivable way all these changes can take place satisfying equation (5) at the same time is by having an increase in  $w$  and the corresponding prosperity spillover effect.

Hence, an increase in  $T$  holding  $K$  constant by a maximizing policy maker will increase wages everywhere, reduce public employment in the poor jurisdiction and increase it in the rich one.



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<sup>7</sup> That is, a maximizing policy maker will be in equilibrium by choosing the values of  $L^G$  and  $N^G$  such that  $\beta > 1$ .

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Findlay, Ronald; "The New Political Economy: Its Explanatory Powers for LDCs" in Gerald M. Meier (editor) Politics and Policy Making in Developing Countries (International Center for Economic Growth, San Francisco, 1991).



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