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Intrinsic Bubbles and Regime Switching

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INTRINSIC BUBBLES AND REGIME SWITCHING

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1. INTRODUCTION

Froot and Obstfeld (1991) explain the persistent deviation of stock prices from those predicted by present value models by the existence of on an "intrinsic bubble"; that is, a bubble with the appealing characteristic of being dependent on economic fundamentals. It is this property which captures the idea that prices might overreact to movements in the fundamentals. However one limitation of their model is that it does not allow for the possibility of regime changes. If these changes do occur, two different issues must be addressed: first, there is the well known result from the rational expectations literature which states that expected future changes in the process that drives dividends should affect the current fundamental prices; second, actual changes in the dividend process should also affect the intrinsic bubble. If breaks are present in the data, their framework should be extended to accommodate the two issues mentioned above. Froot and Obstfeld recognize these potential problems but they do not provide a method to account for them.

In this paper we extend their framework by explicitly allowing for changes in the process that drives dividends. This is done by modeling the fundamentals plus the bubble solution as a function of the current regime. As in Froot and Obstfeld, we assume that the bubble always exists, but we allow it to be a function of the two possible regimes (which differ in the mean and variance of changes in the logarithm of dividends). We find that a model accounting for regime changes appears to provide a better characterization of the evolution of stock prices than the intrinsic bubble model with a single regime. Nevertheless, a model which allows for both intrinsic bubbles and regime switching seems to outperform models that assume either one or the other.

2. DATA ANALYSIS:

A Random Walk Model Versus a Stochastic Segmented Trends and Variances model in the Change in the Logarithm of Real Dividends.

We use data on US stock prices and dividends reproduced by Robert Shiller and listed in Shiller (1989), chapter 26, where more details of the data can be found. The stock prices are January values for the Standard and Poor Composite Stock Price Index. Each observation in the dividend series is an average for the year in question. Nominal stock prices and dividends are deflated by the producer price index (Shiller's series 6 of prices for January each year) to get real stock prices and dividends.

In this section we analyze whether the process that drives the logarithm of real dividends can be characterized as a random walk. Finding evidence that this model is not acceptable, we propose an alternative characterization as a Markov switching model with two states.

Froot and Obstfeld (1991) claim that the hypothesis that the logarithm of real dividends follows a random walk cannot be rejected. They also assume normality of the residuals. This assumption (and the constancy of unconditional moments) is central to the theory developed in their paper. We regress the change in the log of real dividends (d_t) on a constant μ, viz., $\Delta d_t = \mu + \xi_t$, for the period 1900-1987[1]. We find that the assumption that ε_t is normally distributed is violated. A Jarque-Bera test for normality gives a test statistic of 21.5241 which has a $\chi^2(2)$ distribution under the null hypothesis. The errors also appear to contain ARCH effects ($\chi^2(4) = 13.6346$). Figure 1 graphs the data for the change in the logarithm of real dividends. It suggests that during the period from 1952 dividend growth was more stable than in earlier periods. The period between 1915 and 1945 looks particularly volatile.

Since dividends do not appear to be well represented as a random walk with normally distributed innovations, we investigate whether (i) the dividend process can be represented as a GARCH process and (ii) it contains structural breaks and can be represented as a process with changing means and variances. Support for these possibilities is provided by a rolling regression of the rate of expansion of dividends against a constant. Figures 2 and 3 show the rolling regression results. Figure 2 suggests some variation in the rate of growth of dividends. Figure 3 clearly shows a decline in the standard deviation of innovations in the second part of the sample.

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[1] We use the data in Shiller (1989) because it has been very widely used. Froot and Obstfeld (1991) state that, in addition to the data on which their published results were based, they also used this data and found their results with respect to the existence of bubbles to be unaffected.

As is well known in the ARCH literature, structural breaks can cause GARCH models to appear to be IGARCH (see Diebold (1986)) and can also cause a model containing two regimes, each with constant variance to appear as GARCH. We investigate this issue by dividing the sample into two sub-periods, 1900-1947 and 1948-1987 and we find that for each period we cannot reject the null hypothesis of constant variance. Engle's LM test for ARCH effects shows none to be present for either sub-period: $\chi^2(4) = 6.1523$ and $\chi^2(4) = 3.0929$ respectively[2].

This issue can be clearly understood by fitting a GARCH model for the whole sample size. In table 1 we present the results of fitting GARCH for alternative sample sizes. Here we cannot reject the GARCH specification (and the results in fact might be taken to suggest an IGARCH process). Nevertheless, when we divide the sample into two sub-periods, 1900-1947 and 1948-1987 we find no support for the the GARCH specification in any of the sub-periods[3]. Therefore, we do not pursue the GARCH parameterization of the dividend process. In the next section we analyze the mean - variance switching representation.

2.1 A SWITCHING MEAN - VARIANCE REPRESENTATION.

Given the apparent non-constancy of the unconditional moments of the random walk model, we use the discrete regime-switching model proposed by Hamilton (1988) to characterize the first difference of the logarithm of real dividends. This model postulates the existence of an unobserved variable (denoted s_t) which takes the values 1 or 2. When $s_t = 1$, the first difference of the logarithm of real dividends, d_t, is distributed $N(\mu_1,\sigma_1^2)$ and when $s_t = 2$, is distributed $N(\mu_2,\sigma_2^2)$. The states are assumed to follow a first order Markov process with $p(s_t = 1|s_{t-1} = 1) = p$ and $p(s_t = 2|s_{t-1} = 2) = q$.

Table 2 shows the results obtained from estimating this model for the sample

[2] Note that , the variance across periods does not need to be the same. This issue is investigated below.

[3] More formal testing of changes in the unconditional variance within the Garch framework could be carried out. Nevertheless, we feel that not finding ARCH effects in any sub-period is strong enough evidence against any Garch specification.

1900 - 1987. Both standard deviations are significantly different from zero at the 1% level, but the mean is not significantly different from zero in state one. If we compare the results presented in table 2 with Froot and Obstfeld's results, $\mu = 0.011$ and $\sigma = 0.122$, we find that their values seem to be an approximate average of the means and variances in states one and two. We find that state 1 is a low growth/high variance state, and state 2 is a high growth/low variance state. Figure 4 graphs the allocation of observations between the two states. The interwar period is attributed with high, though fluctuating, probability to the low growth/high variance state, state 1, as might have been expected on the basis of visual inspection of the dividend series and the results of the rolling regressions.

Finally, we perform specification tests on the model as in Engel and Hamilton (1990). The tests presented in table 3 are White's (1987) test and Lagrange multiplier specification tests. White's is a score type test: which is based on the fact that if a maximum likelihood model is correctly specified, the score statistics should be serially uncorrelated. Hamilton (1993) extends this test to the Markov switching model[4].

Hamilton recommends, on the basis of Monte-Carlo experiments, that for small samples one might prefer to use the 1 percent critical value from the asymptotic distributions as a guide for a 5 per cent small-sample test based on the Newey-Tauchen-White specification tests or Lagrange multiplier tests.

Table 3 appear to show the presence of autocorrelation, though all other tests for misspecification are passed. Note that the hypothesis of the Markov specification has not been rejected.

3. MODELS OF STOCK PRICES

Froot and Obstfeld (1991) take as their point of departure the familiar condition that the real stock price should equal the present discounted value of the real dividend payment plus the real stock price next period:

$$P_t = e^{-r}E_t(D_t + P_{t+1}) \qquad (1)$$

[4] For more details of the application of these tests see Hamilton (1993).

The real rate of discount r is assumed to be constant (the price P_t might be seen as the start-of-period price, with the dividend D_t paid at the end of the period). The present value solution for P_t, denoted by P_t^{pv} is,

$$P_t^{pv} = \sum_{\tau = t}^{\infty} e^{-r(\tau - t + 1)} E_t(D_\tau) \tag{2}$$

and any bubble in the stock price satisfies,

$$B_t = e^{-r} E_t(B_{t+1}) \tag{3}$$

The process that drives the log of dividends is assumed to be random walk with drift μ.

$$d_{t+1} = \mu + d_t + \xi_{t+1} \tag{4}$$

The "intrinsic bubble" is postulated to be a non-linear function of the dividend D_t which satisfies equation (3). It turns out that the following function satisfies this equation,

$$B(D_t) = c\, D_t^{\lambda} \tag{5}$$

where λ is the positive root of the quadratic equation,

$$\lambda^2 \sigma^2 / 2 + \lambda \mu - r = 0 \tag{6}$$

c is an arbitrary constant, and μ and σ are the conditional mean and standard deviation of Δd_{t+1}, respectively. Substituting equation (5) into equation (4) and using equation (3), it is easy to verify that equation (5) satisfies the bubble solution (see Froot and Obstfeld (1991)).

If we substitute equation (4) into equation (2), we see that the present value is proportional to dividends.

$$P_t^{pv} = \kappa\, D_t \tag{7}$$

where $\kappa = (e^r - e^{(\mu + \frac{1}{2} \sigma^2)})^{-1}$.

Therefore, the price equation can be written as the sum of the present value and the bubble component:

$$P_t = k D_t + c D_t^{\lambda} \tag{8}$$

Because (8) contains both D_t and D_t^{λ} as explanatory variables, the data might be near collinear (at least for values of λ near 1). Froot and Obstfeld divide through by D_t and estimate, both by OLS and ML, the following equation:

$$\frac{P_t}{D_t} = k + c D_t^{\lambda - 1} + \eta_t \tag{8'}$$

Notice that equation (8) has been augmented by a disturbance η_t which they interpret as a random measurement error distributed $N(0,\Theta)$.

Froot and Obstfeld's main conclusion is that the existence of this type of bubble cannot be rejected and it may account for most of the observed difference between the stock prices and the fundamentals. Nevertheless, they interpret these results cautiously and suggest that they could be due to over-identifying restrictions, such as the assumption that the logarithm of real dividends follows a martingale. Estimating this model using Shiller's (1989) data, we reject the null hypothesis that there does not exist an intrinsic bubble. We obtained $k = 15.39$ and $c = 0.034$.

Table 4, columns 3 and 4, gives our estimates of Froot and Obstfeld's model. In column 3, a two stage procedure has been used. The parameters of the process for dividends (μ and σ of equation (4) above) are first estimated from data on dividends and used to compute a value for λ. This is then used in the estimation of the equation for stock prices, equation (8'). In table 4, column 4, both equations (4) and (8') are estimated jointly, with λ determined by equation (6) and k by equation (7). Both sets of estimates display evidence of misspecification.

3.1. ALLOWING FOR REGIME SHIFTS IN THE DIVIDEND PROCESS.

A change in regime which alters the dividend process has two important effects in the model of "intrinsic bubbles". The first, which is of a general nature and common to any model which allows for a bubble, is that bubbles and expected future changes in regimes are observationally equivalent if the econometrician does not explicitly model the regime changes (see Flood and Garber (1980) or Hamilton (1986)).

The second effect is specific to the intrinsic bubble. Since the bubble is a function of the fundamentals, a change in the process driving the fundamentals (dividends) has to change the bubble.

In this section we modify Froot and Obstfeld's model to account for both effects, assuming - as they do - that bubbles are always present. We assume as before, in equation (1), that the price of shares equals the expected present discounted value of the end of period price plus the dividend. The logarithm of the dividend is assumed to follow a random walk with drift, but the drift and the innovation variance depend on the state of the economy. The evolution of the logarithm of real dividends can therefore be written as,

$$d_{t+1} = d_t + \mu_1(2-s_{t+1}) + \mu_2(s_{t+1}-1) + (\sigma_1(2-s_{t+1}) + \sigma_2(s_{t+1}-1))\varepsilon_{t+1} \quad (9)$$

$$s_t = 1,2$$

where s_t is an index for the state of the economy at date t+1, μ_i is the drift in state i, ε_t is a sequence of i.i.d. standard normal variables ($\varepsilon_t \sim N(0,1)$), and σ_i^2 is the innovation variance in state i.

Therefore, the expected value of dividends at t+1, based on information up to and including time t, including knowledge of the state of the system at time t, can be written as

$$E(D_{t+1}|\Omega_t, s_t = 1) = D_t(p\ a_1 + (1-p)\ a_2) \text{ when } s_t = 1,$$

and

$$E(D_{t+1}|\Omega_t, s_t = 2) = D_t((1-q)\ a_1 + q\ a_2) \text{ when } s_t = 2,$$

where $a_1 = e^{\mu_1+\sigma_1^2/2}$ and $a_2 = e^{\mu_2+\sigma_2^2/2}$, and the information set Ω_t includes $(s_t, s_{t-1},\ldots, D_t, D_{t-1},\ldots, P_t, P_{t-1},\ldots)$

The fundamental value of the stock price can be derived in the following way. Suppose that when $s_t = 1$, $P_t = k_1 D_t$, and that when $s_t = 2$, $P_t = k_2 D_t$, for some values of k_1 and k_2. These values satisfy

$$k_1 = e^{-r} (1 + p\, k_1\, a_1 + (1-p)\, k_2\, a_2)$$

and (10)

$$k_2 = e^{-r} (1 + q\, k_2\, a_2 + (1-q)\, k_1\, a_1).$$

The intrinsic bubble in the stock price is modeled as follows. Suppose the bubbles are $B_t = c_1 D_t^{\lambda}$ if $s_t = 1$, and $B_t = c_2 D_t^{\lambda}$ if $s_t = 2$, for some values of λ, c_1, and c_2. The bubble satisfies $B_t = e^{-r} E(B_{t+1} | \Omega_t)$.

Then, when $s_t = 1$, we must have

$$c_1 D_t^{\lambda} = e^{-r}(c_1\, p\, D_t^{\lambda}\, e^{\lambda\mu_1 + \lambda^2\sigma_1^2/2} + (1-p)\, c_2\, D_t^{\lambda}\, e^{\lambda\mu_2 + \lambda^2\sigma_2^2/2})$$

and when $s_t = 2$, we must have

$$c_2 D_t^{\lambda} = e^{-r}(c_2\, q\, D_t^{\lambda}\, e^{\lambda\mu_2 + \lambda^2\sigma_2^2/2} + (1-q)\, c_1\, D_t^{\lambda}\, e^{\lambda\mu_1 + \lambda^2\sigma_1^2/2}).$$

These two equations can be solved for λ and the ratio c_1/c_2. The first equation gives

$$c_1/c_2 = \frac{(1-p)\, e^{\lambda\mu_2 + \lambda^2\sigma_2^2/2}}{(e^r - p\, e^{\lambda\mu_1 + \lambda^2\sigma_1^2/2})} \qquad (11)$$

And the second gives

$$c_1/c_2 = \frac{e^r - q\, e^{\lambda\mu_2 + \lambda^2\sigma_2^2/2}}{(1-q)\, e^{\lambda\mu_1 + \lambda^2\sigma_1^2/2}} \qquad (12)$$

Equations (11) and (12) have a unique positive solution for c_1/c_2 and λ. Equation (11) has $c_1/c_2 < 1$ when $\lambda = 0$, and is increasing in λ, so long as the denominator remains positive. Equation (12) has $c_1/c_2 > 1$ for $\lambda = 0$, and is decreasing in λ, reaching zero when,

$$q \exp\left(\lambda\mu_2 + \lambda^2\sigma_2^2/2 - r\right) = 1.$$

Consequently there is just one combination of λ (> 0) and the ratio c_1/c_2 (> 0) which solves equations (11) and (12). Figure 5 illustrates the solution.

Putting together these results, we can derive a price equation for each state, as the sum of the fundamental component and the bubble component for each state.

$$P_{s_t} = P^{pv}_{s_t} + B_{s_t}(D_t)$$

where $P^{pv}_{s_t} = \left(\kappa_1(2-s_{t+1}) + \kappa_2(s_{t+1}-1) \right) D_t$,

$B_{s_t}(D_t) = \left(c_1(2-s_{t+1}) + c_2(s_{t+1}-1) \right) D_t^{\lambda}$ and $s_t = 1,2.$

Under this assumption, we estimate jointly the following system:

$$\begin{cases} \dfrac{P_t}{D_t} = \kappa_1 + c_1 D_t^{\lambda - 1} + \Theta_1 \upsilon_t \\ d_t = \mu_1 + d_{t-1} + \sigma_1 u_t \end{cases} \qquad \text{In State one.}$$

$$\begin{cases} \dfrac{P_t}{D_t} = \kappa_2 + c_2 D_t^{\lambda - 1} + \Theta_2 \upsilon_t \\ d_t = \mu_2 + d_{t-1} + \sigma_2 u_t \end{cases} \qquad \text{In State two.}$$

u_t and υ_t are independent white noise processes while Θ_1 and Θ_2 are standard deviations. Notice that, as in Froot and Obstfeld (1991) we have augmented the model by random disturbances which may be interpreted as measurement errors. We also allow the variance of these errors to differ between states. We assume that the unobserved states follow a first order Markov process.

The estimation procedure assumes that agents in the financial markets know the actual state of the system, s_t, at each point in time, whereas the econometrician does not, and has to make inferences of it based on the observable history of the system, i.e., the information contained in the history of stock prices and dividends.

We estimate the model subject to the theoretical restrictions on k_1, k_2 and c_1, c_2, λ implied by equation (10), and (11)-(12) respectively. The latter non-linear restrictions cannot be solved analytically and we solve them numerically in the optimizing routine. The filter which is used in the estimation procedure is described briefly in the Appendix.

4. EMPIRICAL RESULTS

We estimate jointly the equations for dividends and stock prices allowing for both bubbles and regime switching as described in section 3, using maximum likelihood estimation. The parameter λ is a function of μ_i, σ_i, p and q, and is obtained from a subroutine that solves equations (11) and (12) numerically. The constant discounting factor is chosen as in Froot and Obstfeld to be the sample - average gross real return r = 0.0816[5]. The estimation strategy and the filter are described briefly in the Appendix.

To carry out specification tests with this model we compute the error terms $\varepsilon_t = pr_t - E(pr_t|I_t;\psi)$, where $pr = (P/D)$ and ψ is the parameter vector. The conditional expectations of pr_t are constructed by multiplying the probabilities of the states obtained from the filter by the functional form in those states. Based on the sample estimates $\hat{\psi}$, the predicted values of pr_t can be written as,

$$E[pr_t|I_t;\psi) = P(s_t = 1|I_t)(\hat{k}_1 + \hat{c}_1 D_t^{\hat{\lambda}-1}) + P(s_t = 2|I_t)(\hat{k}_2 + \hat{c}_2 D_t^{\hat{\lambda}-1})$$

We then standardize the residuals by dividing them by the conditional standard errors. Tests for AR (Godfrey-Breusch) and ARCH (Engle) errors are then performed. In table 4, we show both uncorrected standard errors and also heteroskedasticity- and autoregressive- consistent standard errors[6].

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[5] For a discussion of the appropriateness of this value see Froot and Obstfeld (1991).

[6] ML estimation was carried out using a variable-metric algorithm that

Table 4, column 1, and figure 6 show the results of estimating the model described in section 3.

The estimated means and variances of the logarithm of real dividends are mainly separated as in the univariate filter for dividends reported in table 2. State 1 is a low-mean / high-variance state and state 2 a high-mean / low-variance state. The constants of proportionality between dividends and the fundamental share price are $k_1 = 15.01$ and $k_2 = 17.97$, respectively. Thus it appears that the higher rate of growth of dividends in state 2 is not fully offset by their lower variance in that state, and the fundamental share price is a little higher relative to dividends in that state than in state 1. These results appear to be consistent with rational expectations models in the presence of regime changes. The bubble coefficient is significant and is higher in state 2 ($c_2 = 0.29$) than in state 1 ($c_1 = 0.15$). The elasticity of the bubble with respect to the dividend is 2.14. The model shows no sign of ARCH errors of up to order 4. There is evidence of autoregressive errors, but the standard errors have been constructed consistent with this.

Figure 6 shows the evolution of stock prices, the fundamental stock price, and the fundamental-plus-bubble stock price predicted by the model. The bubble term accounts for most of the large deviations between fundamentals and stock prices, especially those in the second part of the sample where the bubble term appears to be particularly important.

The allocation of time periods to the two regimes is shown in figure 7. The period between 1900 and 1910 is attributed with high probability to regime 2. The period between 1910 and 1955 is attributed mostly to regime 1, with brief departures from it around 1930 and 1945. The period from 1955 to 1975 is attributed to the high growth/low variance regime (state 2). The remaining observations fall in regime 1, with a final shift before the crash in 1987 to regime 2. The periods for which observations are attributed to regime 2 are ones in which the actual share price appears to be a long way above the fundamental price and the bubble component is particularly large.

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approximates the Hessian according to the Broyden-Fletcher-Goldfard-Shanno update. The pre-whitened quadratic spectral kernel with data-dependent bandwidth discussed in Andrews (1991) and Andrews and Monahan (1992) was used for the covariance matrix estimator.

Table 4, column 2, shows the results of imposing the assumption of no bubble in stock prices, an hypothesis strongly rejected: the likelihood ratio test statistic is 33.52, distributed $\chi^2(1)$. The restricted model shows considerable evidence of mis-specification. Figure 8 shows estimated stock prices under the assumption of no bubble.

Table 4 column 3 present the results of a two-step estimation procedure. In the first step, an estimate of the dividends process is used to derive values for μ and σ, which are inserted into equation (6) to generate a value for λ. In the second step, the equation for stock prices is estimated using the value of λ derived from step 1; k is estimated freely. In table 4, column 4, the two equations are estimated jointly subject to all the restrictions imposed by equations (6) and (7) on λ and k. These two different estimation procedures give results which are very similar.

Comparing the model with both bubbles and regime switching (table 4, column 1) with the model with bubbles only (table 4, columns 3 and 4) we see that the models without regime-switching show much more evidence of mis-specification: there is evidence of AR and ARCH errors. Also the likelihood is much lower in columns 3 and 4 than in column 1: -33 as against 0. Although formal tests are difficult to carry out, this is strongly indicative of the importance of allowing for regime-switching.

Figure 9 shows estimated stock prices allowing only for bubbles, with no regime-switching. Comparing figure 9 with figure 6, it is evident that the assumption of no regime-switching causes a substantial deterioration of the fit of the model, especially in the period from 1950. A comparison of figures 8 and 6 shows that the assumption of no bubbles causes a smaller deterioration in the fit. A comparison of figures 8 and 9 reinforces the view that allowing for regime-switching in the dividends process contributes at least as much to an explanation of the data as does allowing for the bubble.

5. CONCLUSIONS

In this paper we have extended Froot and Obstfeld's formulation of intrinsic bubbles to allow for the existence of regime changes in the process driving logarithm of real dividends. Since structural breaks appear to be present in the dividend process, we modeled dividends as a process with two states, each with a separate mean and variance. To model stock prices, we considered the solution consisting of the fundamental plus bubble, consistent with the two-state dividend process. We found that this model appeared to explain the behaviour of US stock prices between 1900 and 1987 better than a model which for an intrinsic bubble but that made no allowance for regime shifts.

When stochastic regime-switching is introduced, part of the fluctuations of stock prices that would otherwise have been interpreted as a bubble may now be interpreted as shifts in the fundamental price resulting from a change of regime. Our results suggest that the fluctuations in stock prices explicable as an intrinsic bubble can instead be well explained in terms of regime-switching. Nevertheless a model that allows for both regime switching and intrinsic bubbles clearly outperforms the simple regime switching model, favouring the existence of regime switching intrinsic bubbles.

We interpret the intrinsic bubble as an empirical possibility alongside other deviations from fundamentals such as fads or time-varying discount factors. Further, we note that the incorporation of an intrinsic bubble into the stock price when there are stochastic regime switches, requires a modification of the original framework proposed by Froot and Obstfeld.

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Table 1

	$d_t = \mu + d_{t-1} + u_t$;	$\sigma^2_t = \alpha + \beta\, u^2_{t-1} + \gamma\, \sigma^2_{t-1}$	
	1900-1987	1900-19471	1948-1987
μ	0.0130 (0.0101)	0.0068 (0.0216)	0.0094 (0.0125)
α	0.0005 (0.0004)	0.0251 (0.0021)	0.0066 (0.0015)
β	0.2875 (0.0563)	0.0374 (0.0891)	0.2500 (0.1471)
γ	0.7200 (0.0543)	0.0632 (0.1482)	0.0360 (0.1344)
Log lik	142.9681	60.8347	88.8523

The values between brackets are standard deviations.

Table 2 Estimates of a Regime-Switching model for the dividend process

μ_1	0.0007 (0.0334)
μ_2	0.0223 (0.0113)
σ_1 =	0.1997 (0.0319)
σ_2	0.0546 (0.0134)
p	0.8898 (0.0984)
q	0.8410 (0.1792)
Log lik.	151.2649

The values between brackets are standard deviations.

Table 3

Specification Tests

Regime-Switching Model for the Dividend Process

White test for Autocorrelation ($\chi^2(4)$)	13.4670	**
White test for ARCH ($\chi^2(4)$)	7.5263	
White test of Markov Specification ($\chi^2(4)$)	1.8763	
LM Test for Autocorrelation in Regime 1 ($\chi^2(1)$)	0.2478	
LM Test for Autocorrelation in Regime 2 ($\chi^2(1)$)	6.6605	**
LM Test for Autocorrelation across Regimes ($\chi^2(1)$)	6.5647	
LM Test for ARCH in Regime 1 ($\chi^2(1)$)	3.0239	
LM Test for ARCH in Regime 2 ($\chi^2(1)$)	1.3494	
LM Test for ARCH across Regimes ($\chi^2(1)$)	0.0040	

** Significant at 1%

Table 4 Estimation of the joint model for dividends and stock prices allowing for both intrinsic bubbles and regime-switching

	Allowing for Regime-Switching in Dividends: Bubble	No Bubble		No Regime-Switching: Two-step	Joint estimation
μ_1	0.0076	0.0128			
	(0.0054) [0.0077]	(0.0179) [0.0147]	μ	0.0141	0.0115
μ_2	0.0384	0.0651		(0.0101)	(0.0031)
	(0.0040) [0.0086]	(0.0219) [0.0081]		[0.0163]	[0.0093]
σ_1 =	0.1526	0.1469			
	(0.0129) [0.0256]	(0.0123) [0.0231]	σ	0.1338	0.1328
σ_2	0.0636	0.0694		(0.0105)	(0.0100)
	(0.0191) [0.0312]	(0.0172) [0.0291]		[0.0194]	[0.0163]
Θ_1 =	2.9476	3.4156			
	(0.2994) [0.8712]	(0.1769) [0.4191]	Θ	4.0266	4.0428
Θ_2	2.2506	1.9240		(0.3031)	(0.2856)
	(0.3532) [0.5522]	(0.1770) [0.9685]		[0.5030]	[0.5051]
k_1	15.0148	19.3796			
k_2	17.9737	30.6301			
			k	15.3868 (0.8232) [2.1475]	14.3958
p	0.9636 (0.0622) [0.0912]	0.9704 (0.0566) [0.0634]			
q	0.9823 (0.0328) [0.0520]	0.9782 (0.028) [0.0515]			
c_1	0.1529				
	(0.0506) [0.0733]		c	0.0339 (0.0039) [0.0131]	0.0805 (0.0012) [0.0611]
c_2	0.2874				
λ	2.1488			2.78	2.5411
Log lik	0.6475	-16.1194		-33.5911	-33.9658
ARCH(1)	1.7428	19.2565		22.2035	21.9512
ARCH(4)	6.2686	19.9846		22.3975	22.2223
AR(1)	10.908	30.8599		49.2618	49.1687
AR(4)	13.628	33.0537		50.6311	50.5677
RMSE	46.614	62.9099		80.9254	81.3205
MAE	5.9685	6.89884		7.72609	7.74949

The values between parentheses are uncorrected std. errors, those in brackets are autoregressive heteroskedasticity Andrews' consistent std. errors..

APPENDIX

In this appendix we describe the filter used in the estimation procedure described in section 3.1.

The filter begins by generating, from the product of (i) the posterior probability of the state of the system at time t-1 based on information up to time t-1, $P(s_{t-1}|y_{t-1},...,y_0)$, (ii) the ex ante probability of the state at time t conditional on information up to time t-1, $P(s_t|s_{t-1})$.

$$\text{(A1)} \qquad P(s_t,s_{t-1}|y_{t-1},...y_0) = P(s_t|s_{t-1})\ P(s_{t-1}|y_{t-1},...y_0)$$

Marginalizing with respect to s_{t-1} we get.

$$\text{(A2)} \qquad P(s_t|y_{t-1},...y_0) = \sum_{s_{t-1}=0}^{1} P(s_t,s_{t-1}|y_{t-1},...,y_0)$$

The product of this expression and the conditional probability of the observations y_t conditional on the state s_t and the past observations, $P(y_t|s_t,y_{t-1},..,y_0)$, gives the joint probability of the observations y_t and the state s_t, conditional on the observed history of the data.

$$\text{(A3)} \qquad P(y_t,s_t|y_{t-1},...,y_0) = P(y_t|s_t,y_{t-1},...,y_0).P(s_t|y_{t-1},....,y_0)$$

The density of the data y_t conditional on the state s_t and the history of the system can be written as:

$$P(y_t|s_t,y_{t-1},....,y_0) = \frac{1}{(2\pi)^{.5}\,\sigma_{s_t}} \exp(-\ (2\ \sigma^2_{s_t})^{-1}\ (\ \Delta d_t - \mu_{s_t}\ )^2)\ .$$

$$\frac{1}{(2\pi)^{.5}\,\Theta_{s_t}} \quad \exp(-\ (2\ \Theta^2_{s_t})^{-1}\ (\ pr_t - k_{s_t} - c_{s_t} D_t^{\lambda-1}\ )^2)$$

where y_t is a 2x1 vector containing Δd_t and pr_t(the price dividend ratio).

A distinctive feature of this filter is that both the price and the dividend equation depend on the state. Also note that k_1 and k_2 satisfy the system described in equation (10), which may easily be solved algebraically. The restriction between c_1 and c_2 imposed by the theory can only be solved numerically. The program calls a subroutine that solves numerically equations (11) and (12), so each line search is assured to satisfy the restrictions imposed by the model.

From this we can write the density of the data, not conditioned on the state as

$$\text{(A4)}\quad P(y_t|y_{t-1},..,.y_0) = \sum_{s_t=0}^{1} P(y_t,s_t|y_{t-1},...,y_0)$$

and from this we can obtain a posterior for the current unobserved state as

$$\text{(A5)}\quad P(s_t|y_t,y_{t-1},..,y_0) = \frac{P(y_t,s_t|y_{t-1},....,y_0)}{P(y_t|y_{t-1},....y_0)}$$

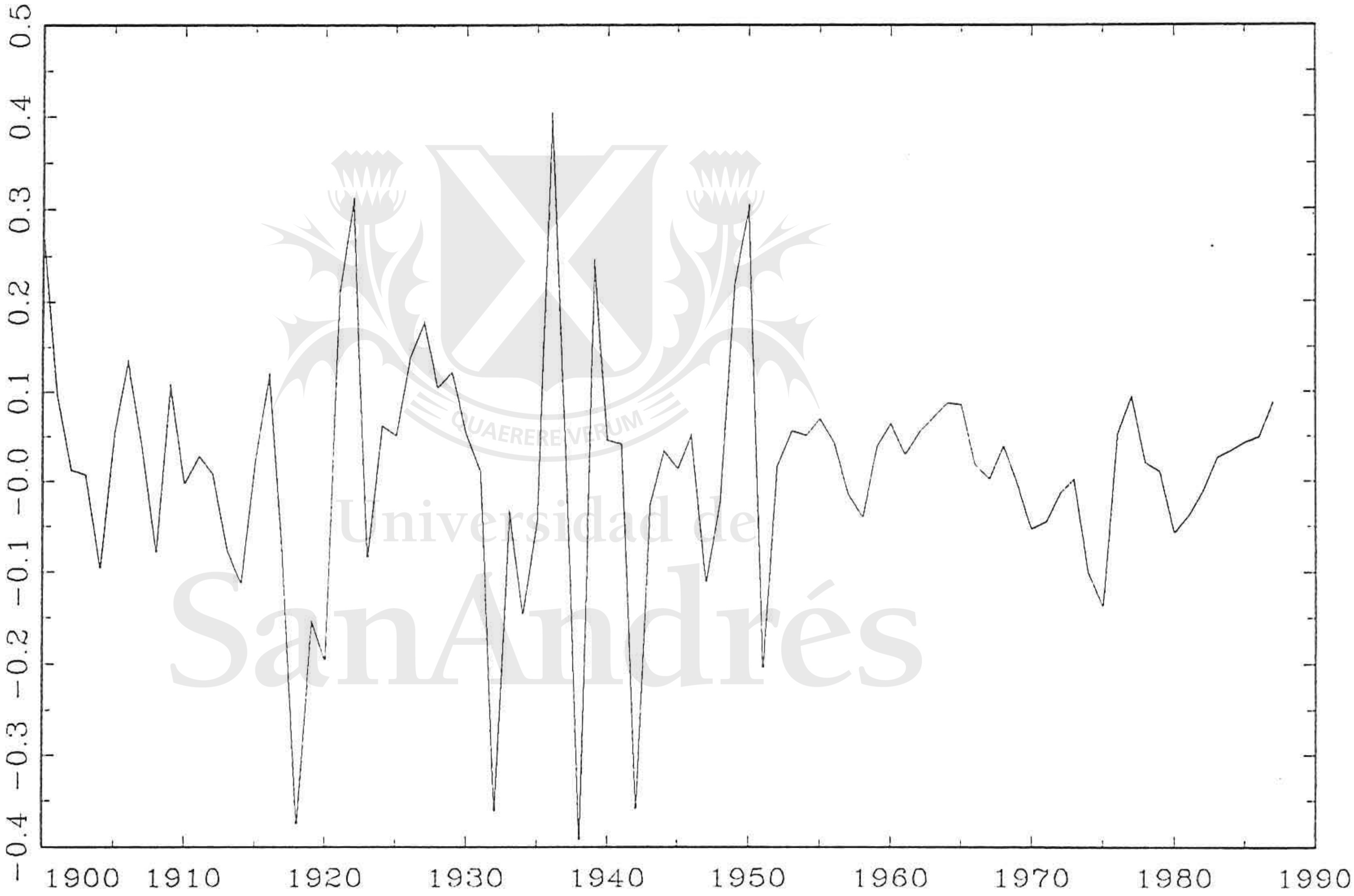
The output from (A5) provides the input into the filter for the next period of time in equation (A1). The estimation process uses the likelihood function for each observation obtained in (A4) above, to generate a likelihood function for the whole sample, which is maximized with respect to parameter values.

To start the filter, at the beginning of the sample, we use $P(s_0)$, which is chosen as the equilibrium Markov unconditional probabilities, as a proxy for $P(s_{t-1}|y_{t-1},.,y_0)$ which is needed in (A1).

Figure 1

GAUSS Mon Jun 5 15:38:56 1995

Growth of Real Dividends



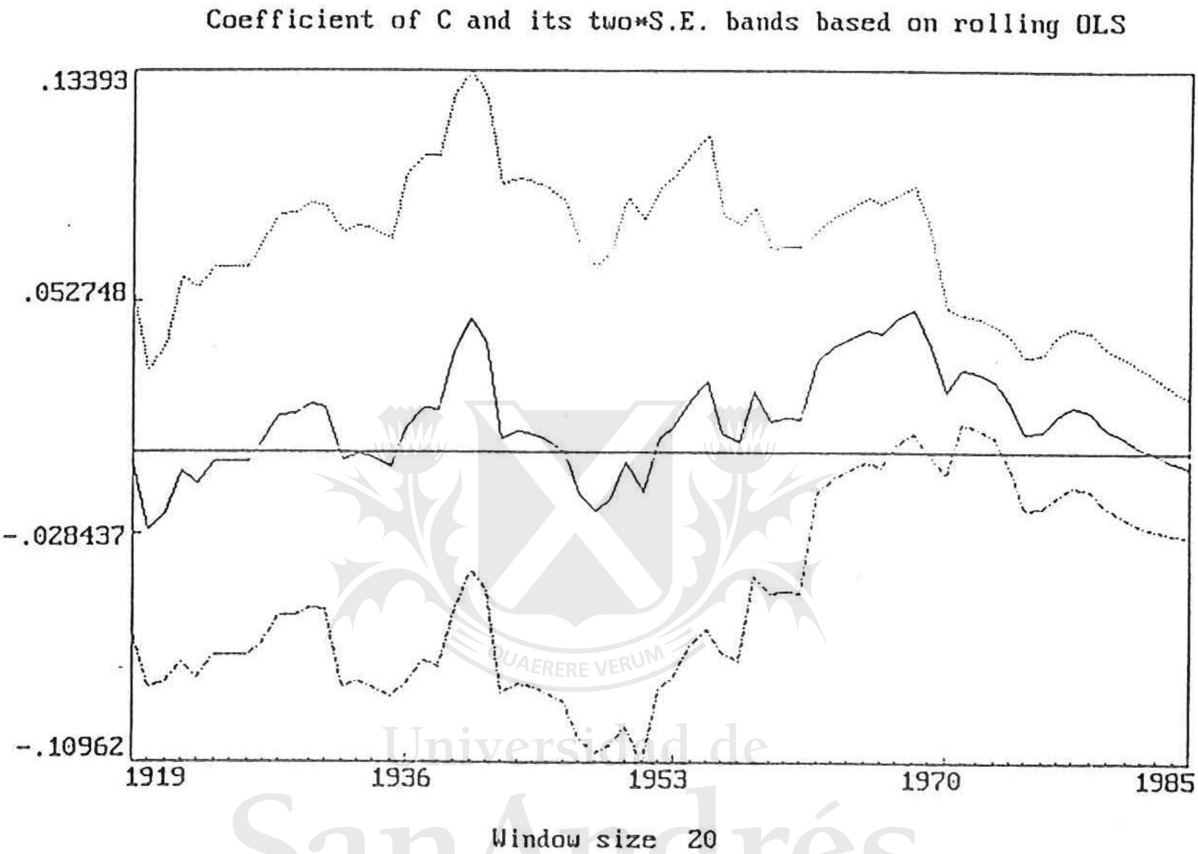

Coefficient of C and its two*S.E. bands based on rolling OLS
.13393
.052748
-.028437
-.10962
1919
1936
1953
1970
1985
Window size 20

Figure 2

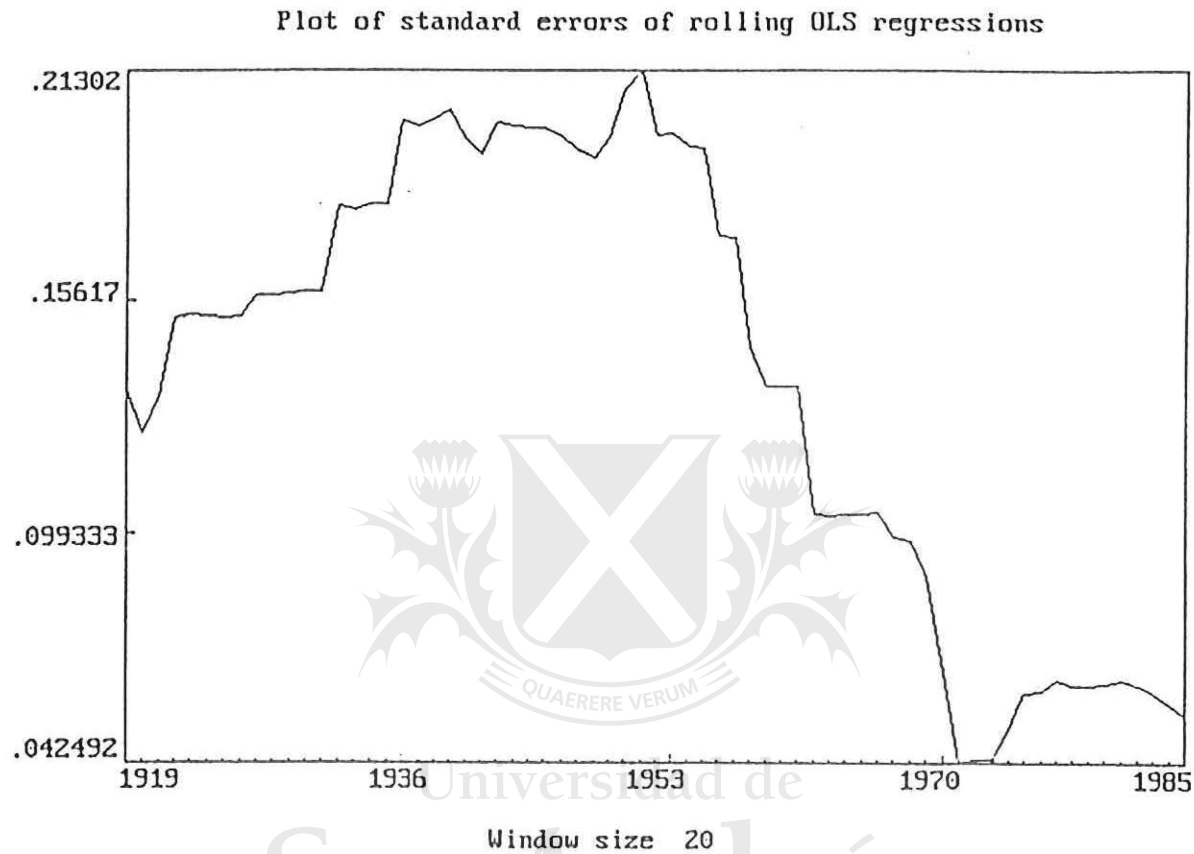

Plot of standard errors of rolling OLS regressions
.21302
.15617
.099333
.042492
1919
1936
1953
1970
1985
Window size 20

Figure 3

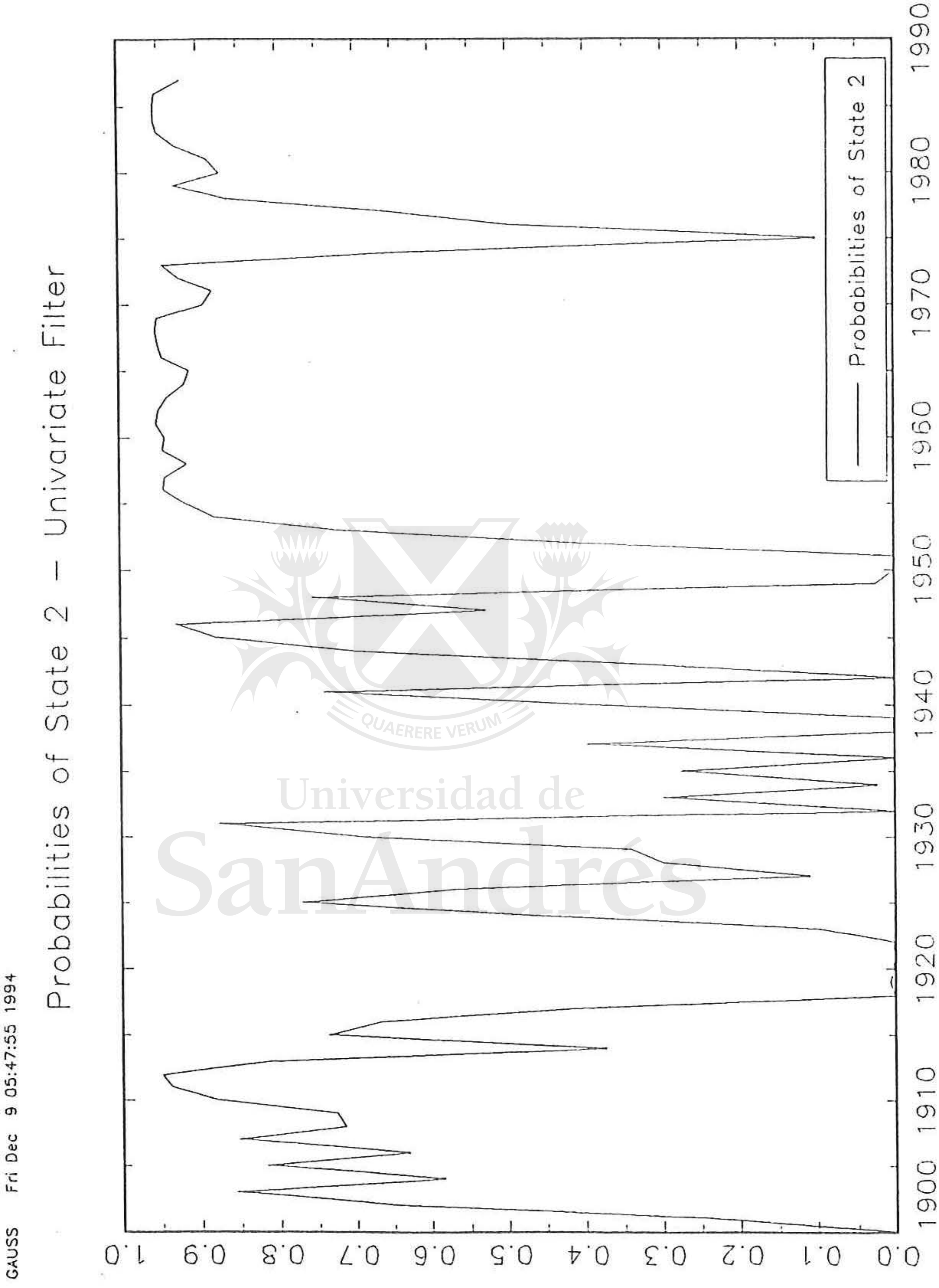
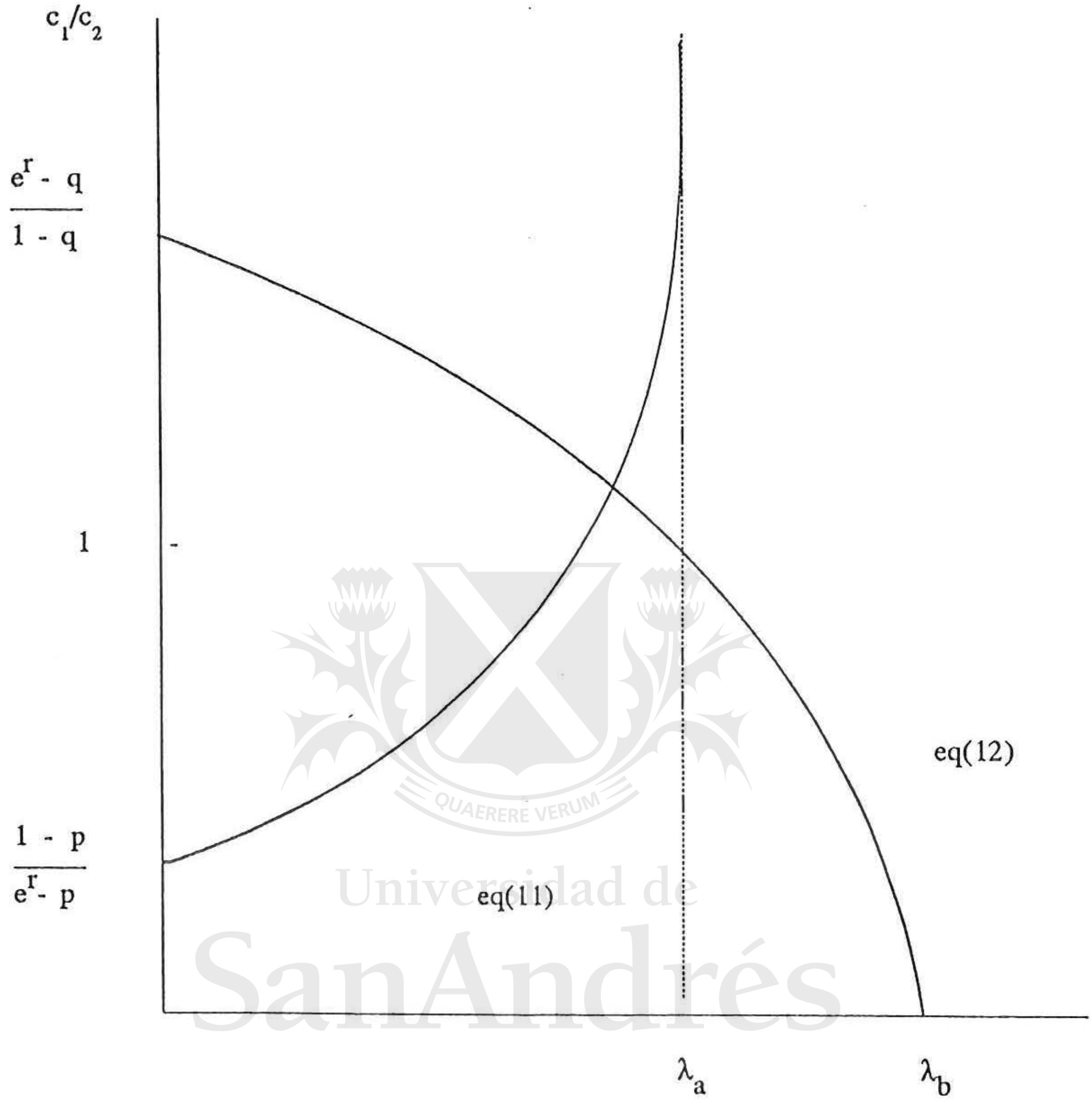

GAUSS Fri Dec 9 05:47:55 1994
Probabilities of State 2 - Univariate Filter
Probablilities of State 2
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 4

The Determination of c_1/c_2 From Equations 11 and 12.



$$\lambda_a = \frac{-\mu_1 \pm \sqrt{\mu_1^2 + 2\sigma_1^2 (r - \log(p))}}{2\sigma_1^2}$$

$$\lambda_b = \frac{-\mu_2 \pm \sqrt{\mu_2^2 + 2\sigma_2^2 (r - \log(q))}}{2\sigma_2^2}$$

Figure 5.

Figure 6

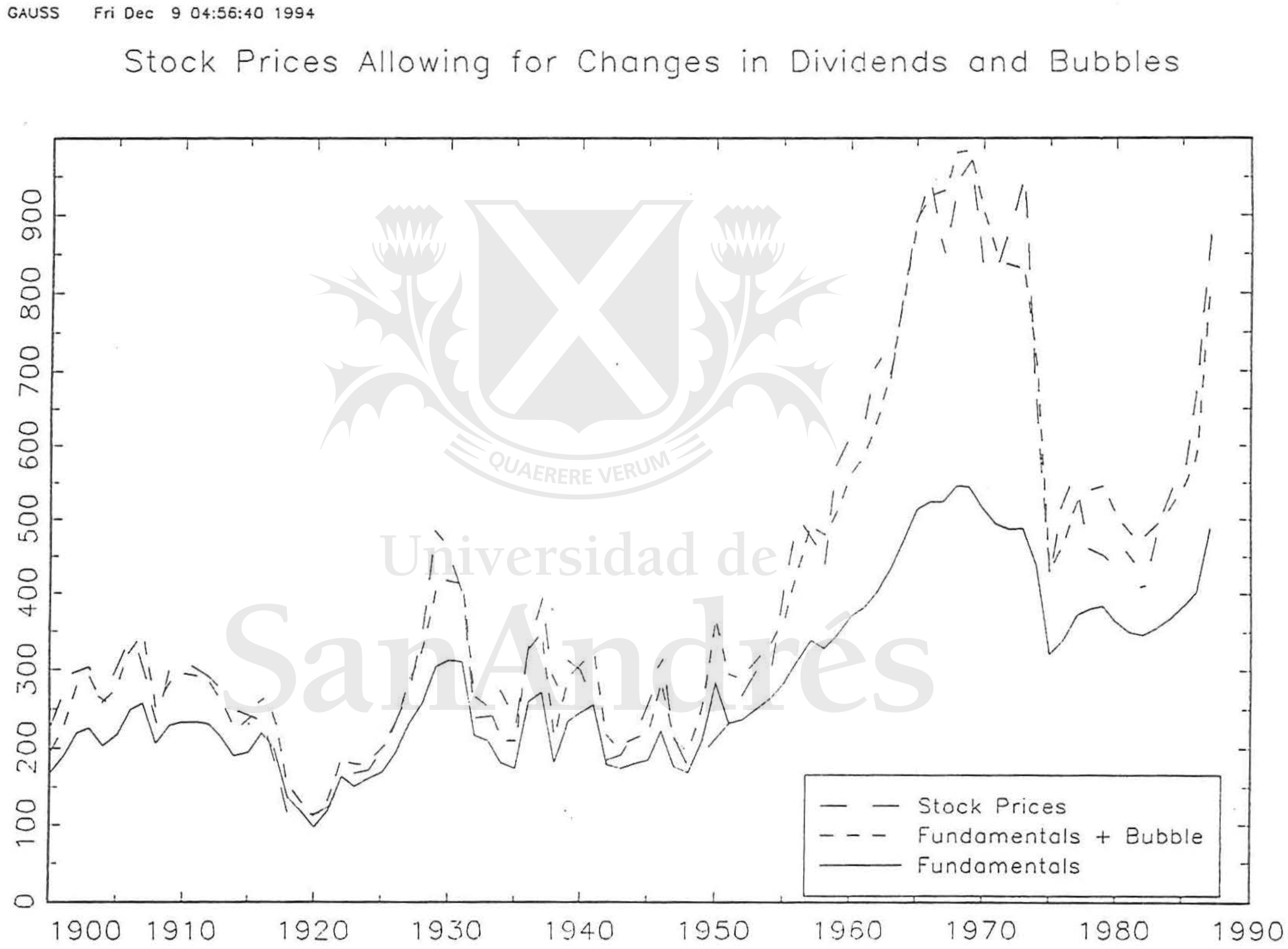

GAUSS Fri Dec 9 04:56:40 1994
Stock Prices Allowing for Changes in Dividends and Bubbles
0
100
200
300
400
500
600
700
800
900
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
Stock Prices
Fundamentals + Bubble
Fundamentals

Figure 7

GAUSS Fri Dec 9 05:01:18 1994

Probabilities of State 2 Allowing for Changes in Dividends and Bubbles

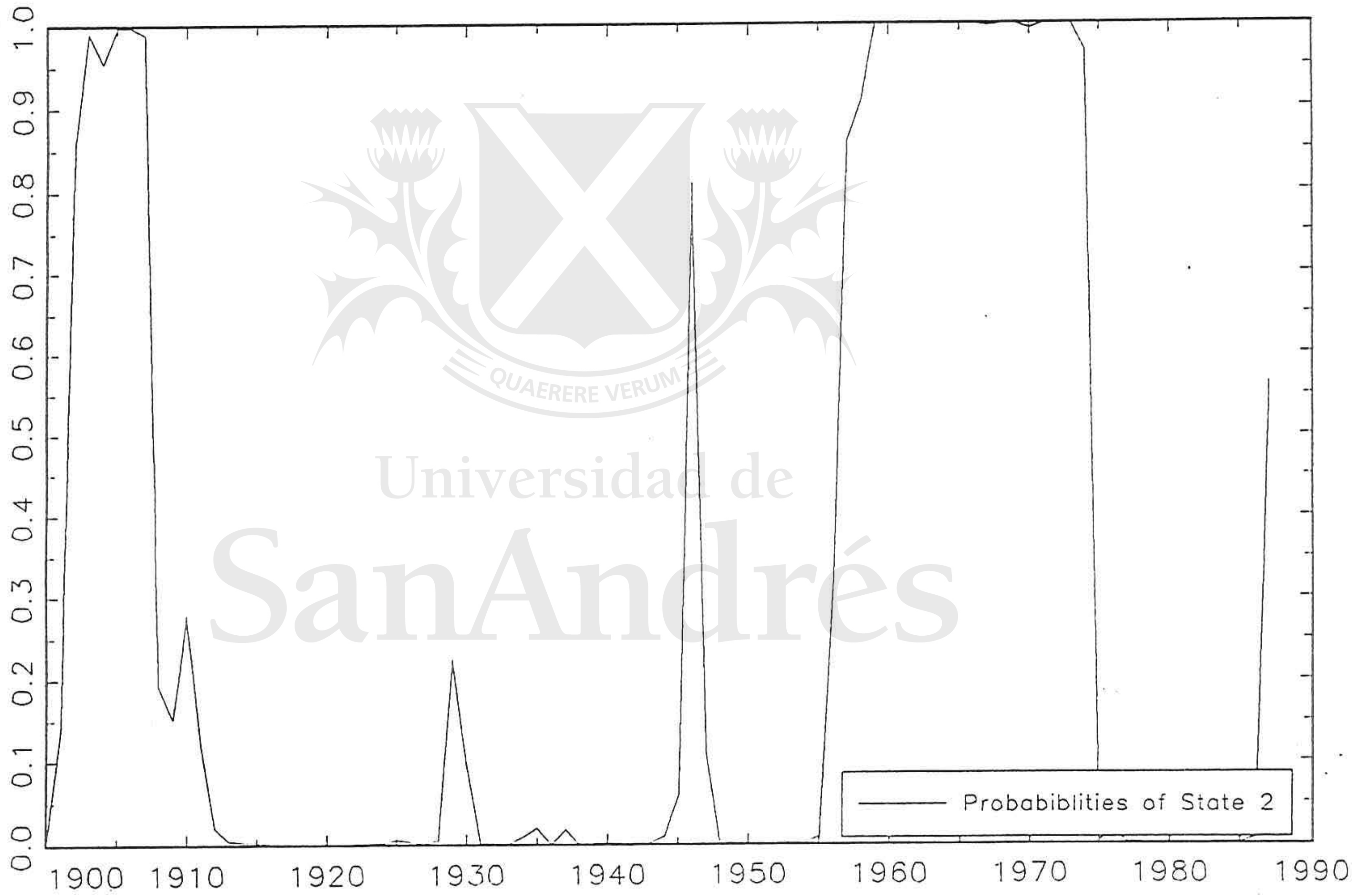


Figure 8

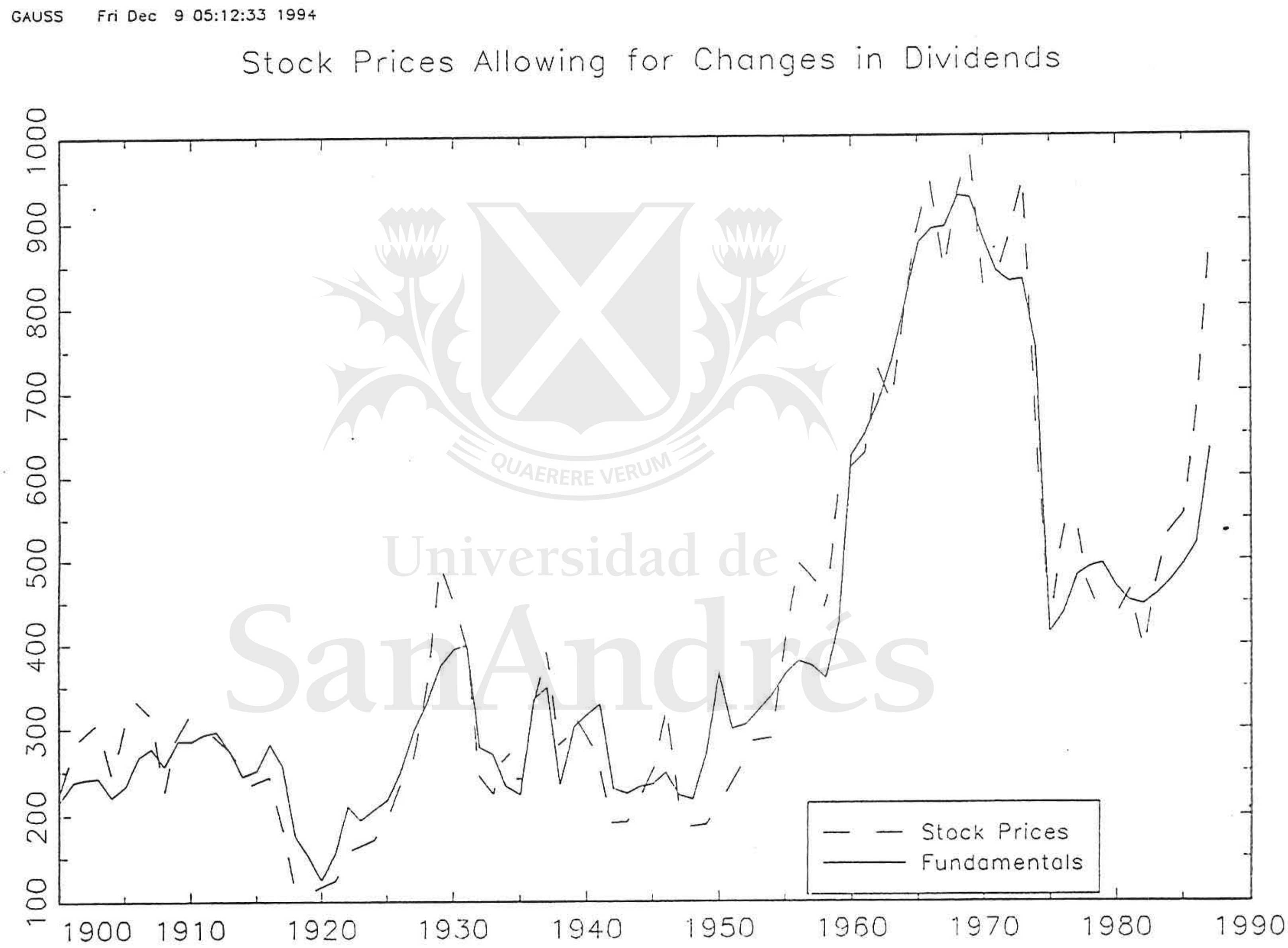

GAUSS Fri Dec 9 05:12:33 1994
Stock Prices Allowing for Changes in Dividends
100
200
300
400
500
600
700
800
900
1000
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
Stock Prices
Fundamentals

Figure 9

GAUSS Fri Dec 9 05:16:43 1994

Stock Prices